General recommendations

i) always one example & counter-example per definition / Theorem
ii) try to remember the strategy of the proofs
iii) use the lecture's notation

Chapter I

i) def.: open, closed, compact, connected set

Chapter II

i) def.: holomorphic functions, (smooth) curves, line integral, primitive
ii) rules for computing derivatives
iii) Cauchy-Riemann equations
iv) convergent power series define holomorphic functions

Chapter III

i) Cauchy's integral formula for a circle
ii) holomorphic functions in a convex, open set have a primitive (+ Goursat's theorem)

Chapter IV

i) def.: infinite products of complex numbers
ii) Theorems: holomorphicity = analyticity (in a disc); Liouville's theorem; analytic continuation; complex parameter integral theorem; zeros of non-const. fct. on a connected set are isolated; convergence theorem for locally uniformly, holomorphic fct.

iii) $f^{(n)}(z_0) = \frac{n!}{2i\pi} \int_C \frac{f(w)}{(w-z_0)^{n+1}} dw$

iv) Cauchy inequalities : $|f^{(n)}(z_0)| \leq n! \frac{\sup_{|w-z_0|=r} |f(w)|}{r^n}$

Chapter V

i) def.: poles, order of poles, principal part & residue of meromorphic fct. at a point, meromorphic fct, $\overline{\mathbb{C}}$, singularities (+ types)

ii) theorems: $f \in \mathcal{H}(U - \{z_0\})$ bounded on $D_r^{\circ}(z_0) \Rightarrow z_0$ removable $\wedge$ f has analytic continuation at z_0; residue formula:

$\int_\gamma f(z) dz = 2i\pi \sum_{z \in \mathring{\gamma}} res_z(f)$ [$\mathring{\gamma}$:= "interior" of γ];

$\frac{1}{2i\pi} \int_\gamma \frac{f'(z)}{f(z)} dz = \#$ zeros $- \#$ poles inside γ (with multiplicity!);

Rouché's theorem, open image theorem, maximum modulus principle

Chapter VI

i) def.: infinite products of complex numbers

ii) $\sum_n |a_n(z)|$ converges locally uniformly $\Rightarrow \prod_n (1+a_n(z))$ holomorphic & in particular convergent

Chapter VII

i) def.: homotopies, simply-connected open sets, (principal) branch of the logarithm on open set, z^α for z in simply-connected set, n-th root of z, winding number

ii) theorems: homotopy theorem; existence of a primitive in s.-c. sets for $f \in \mathcal{H}(U)$; Cauchy's formula in s.-c. set, existence of a branch of the logarithm in s.-c. set; residue formula with winding numbers:

$$\int_\gamma f(z)\,dz = 2i\pi \sum_{z \in S_f} \mathrm{res}_z(f)\, w_\gamma(z) \quad [S_f = \text{set of singularities of domain of } f]$$

Chapter VIII

i) def.: conformal map, conformal equivalence, automorphism

ii) Theorems: conformal maps have non-zero derivative everywhere; Riemann's Mapping Theorem; Schwarz's Lemma; Montel's Theorem; $(f_n)_n$ injective, holomorphic, locally uniformly convergent $\Rightarrow \lim_{n \to \infty} f_n$ is const. or injective